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Accelerated Quantum Bouncer

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Abstract

Quantum Localization comes as a seemingly paradoxical result in a model which is classically described by the Standard Map. The paradox disappears upon realizing that the variables entering the Standard Map are not a conjugate pair for the model. In order to properly quantize the model one must resort to correct conjugate variables, which lead to a different map. We have here a clear illustration that quantizing classical area preserving maps is not a uniquely defined process. In this paper we describe the "parameters of transition"from Standard Map into the more quantum mechanical map. And also show the dependence of "$\triangle P$ verses "$\hbar$", here "$\hbar$"is a dimensionless Plank's Constant.

Keywords: Solar energy resources, Photovoltaic energy prediction, Grid-connected power plant, Simulation.

1 Introduction

A classical system subjected to time-periodic modulation, in general becomes globally chaotic for increasing modulation strengths, and it may indefinitely absorb energy from the external field in a diffusive way. However, in corresponding quantum domain the "diffusive absorption" is suppressed by the quantum interference effect, which is the manifestation of dynamical localization phenomenon, analogous to Anderson localization of solid state physics. The phenomena has been discussed in model systems in quantum chaos, such as, kicked rotator [2], quantum bouncer [3], and molecular systems in

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the presence of electric and magnetic fields [4]. Dynamical localization is a very general phenomenon in periodically driven systems, but it will not occur under conditions of resonance or in special cases distinguished by some form of translational invariance [5, 6]. The "delocalization" in such cases is a purely quantum effect: the longtime unbounded propagation is not related to the corresponding classical diffusion [7, 10].

The kicked rotor is a standard system used in the investigation of classical Hamiltonian chaos and its manifestations in quantum mechanical systems [8]. The motion is classically described by the Standard Map. The size of the stochasticity in the phase space of the map increases with the driving strength, and when the latter is sufficiently strong unbounded diffusion in action space takes place. Important deviations are however found for some values of the driving strength [14], due to the onset of so-called accelerator modes, that produce linear, rather than diffusive, growth of momentum along orbits in a set of positive measure [10].

In recent experiments [11,12], atoms were driven in the vertical direction, and gravity was found to produce remarkable effects. A fraction of the atoms are steadily accelerated, at a rate which is faster or slower than the gravitational acceleration depending on what side of the resonance the driving frequency is. Such atoms are exempt from the diffusive spread that takes place for the other atoms, and their acceleration depends on the difference between the driving and the resonant frequencies [8]. In ref [13] a physical explanation is given, and it is stressed that the phenomenon resembles the accelerator modes in the Standard Map. The accelerating parts of the distributions were hence termed "quantum accelerator modes," at once emphasizing that resemblance, and their purely quantal nature; in fact, they have no classical counterpart in the classical dynamics of the kicked particles in gravity [14]. In this paper, we discuss the "quantum accelerator mapping", that is the transition of Standard Mapping into conjugate pair mapping, which we say quantum mapping and latter we explain corresponding numerical results.

In sec II, we describe the physical system and its Hamiltonian. In sec III, we derive quantum mechanical form of conjugate pair mapping and also discuss the transition parameters of Standard Mapping Hamiltonian into Quantum Mapping Hamiltonian, and also we discuss the Fokker-Plank equation for the given system. In sec IV, we discuss the Mapping suitable to explain the accelerated dynamics.

2 The Experimental System

Thirty years after the first suggestion of Fermi, $Pustyl\acute{n}ikov$ provided detailed study of another accelerator model, which is presently called as quantum bouncer [15, 16] or Fermi accelerator. In his work, $Pustyl\acute{n}ikov$ proved that a particle bouncing in the accelerator system attains modes, where it ever gets unbounded acceleration. This feature

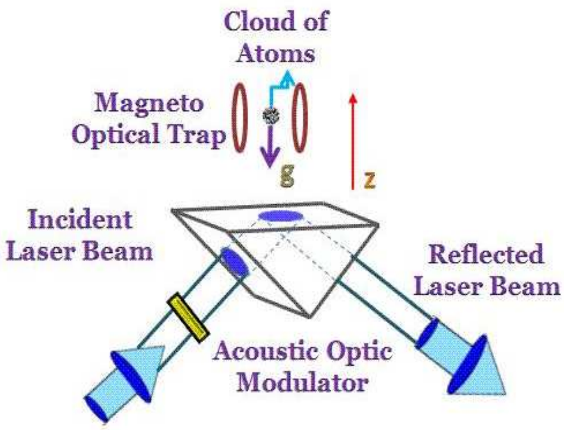

Figure 1.1: A cloud of atoms are trapped and could in a megnateo-optical trap for a few micro-kelvin. And MOT is placed at some height when some one want to start the experiment, it switch off the MOT and atoms are going down with constant acceleration towards the exponential decaying field.

makes the Fermi-$Pustyl\acute{n}ikov$ model richer in its dynamical beauty. In the atomic Fermi accelerator, an atom moves under the influence of gravitational field towards an atomic mirror made up of an evanescent wave field. The atomic mirror is provided a spatial modulation by means of an acousto-optic modulator which provides intensity modulation to the incident laser light field [3]. Hence, the ultra cold two-level atom, after a normal incidence with the modulated atomic mirror, bounces off and travels in the gravitational field, as shown in Fig. 1. In order to avoid any atomic momentum along the plane of the mirror the laser light which undergoes total internal reflection, is reflected back. Therefore, we find a standing wave in the plane of the mirror which avoids any specular reflection [17]. The periodic modulation in the intensity of the evanescent wave optical field may lead to the spatial modulation of the atomic mirror as

$$\bar{I}(\bar{z},t) = I_0 e^{-2\kappa\bar{z}+\epsilon\cos(\bar{\omega}\bar{t})}. \tag{2.1}$$

Thus, the motion of the atom in z-direction follows effectively the Hamiltonian,

$$\bar{H} = \frac{\bar{p_z}^2}{2m} + mg\bar{z} + \hbar\Omega_{eff} e^{-2\kappa\bar{z}+\epsilon\cos(\bar{\omega}\bar{t})}. \tag{2.2}$$

where, Ω_{eff} denotes the effective Rabi frequency. Moreover, ϵ and $\bar{\omega}$ express the amplitude and the frequency of the external modulation, respectively.

3 Quantum Mapping

In case the decay constant "κ"of the evanescent wave field is large, simplified Hamiltonian of our system becomes,

$$\bar{H} = \frac{\bar{p_z}^2}{2m} + mg\bar{z} + V\bar{z}\cos\bar{\omega}\bar{t}. \qquad z \geq 0 \tag{3.1}$$

Here, "$V = \frac{\hbar\omega^2\Omega}{4mg^2}$"potential of the external field. We proceed by introducing the dimensionless position and momentum coordinates, that is, "$p = \frac{\bar{p}}{\sqrt{m\hbar\Omega}}$", "$z = \bar{z}\sqrt{\frac{m\Omega}{\hbar}}$", "$\omega = \frac{\bar{\omega}}{\Omega}$", and "$t = \Omega\bar{t}$". Hence, the Hamiltonian takes the dimensionless form as, that is,

$$H(z,p,t) = \frac{p^2}{2} + \epsilon_0 z + \epsilon z\cos\omega t. \qquad z \geq 0 \tag{3.2}$$

where "$\epsilon_0 = g\sqrt{\frac{m}{\hbar\Omega^3}}$", is the constant field strength, "$\epsilon = V\sqrt{\frac{1}{\hbar m\Omega^3}}$", and "$\omega$", are the oscillating field strength and frequency respectively. The Hamiltonian given in (3.2) describes a particle mass "m"bouncing on an oscillating hard wall in the presence of a gravitational field. This Hamiltonian is integrable in the absence of the time dependent term. We express the time development of the particle moving in time dependent system by the impact map which gives the evolution from immediately after a bounce to immediately after the next bounce [8], that is,

$$\begin{cases} p_{i+1} = -p_i + \epsilon_0 \triangle t_i + \frac{2\epsilon}{\omega}\sin(\omega \triangle t_i), \\ \phi_{i+1} = \phi_i + \omega \triangle t_i. \end{cases} \tag{3.3}$$

The mapping given in Eq. (3.3) takes a simple form when the amplitude of the motion between subsequent kicks is much larger then amplitude of wall oscillations,

$$\begin{cases} p_{i+1} = -p_i + \frac{2\epsilon}{\omega}\sin(\omega \triangle t_i), \\ \phi_{i+1} = \phi_i + \frac{2\omega}{\epsilon_0} p_{i+1}. \end{cases} \tag{3.4}$$

The map obtained in Eq. (3.4) is the Standard Map. The onset of diffusive excitation in the system as the chaos parameter "$K = \frac{4\epsilon}{\epsilon_0}$"takes a value larger than 0.96, i.e., when the perturbation amplitude exceeds the critical value "$\epsilon_{cr} = \frac{\epsilon_0}{4} = 0.96$" [18]. The fact that the Standard Map provides a good description of the classical model does not command any similarity of the quantum model with the quantum Kicked Rotator (which is a quantization of the classical Standard Map). As a matter of fact, the variables "p"and "ϕ"in Eq. (3.4) are not a conjugate pair in the full Hamiltonian formulation of the model. The correct conjugate variable to the field phase "ϕ"is the quantity "$N = \frac{E}{\omega}$", where E is unperturbed energy and its value can be determined as,

$$E = \frac{(3\pi\epsilon_0 I)^{\frac{2}{3}}}{2}. \tag{3.5}$$

In the conjugate variables "(N, ϕ)"the impact mapping given in Eq. (3.4), on neglecting second order terms in "ϵ", takes the form,

$$\begin{cases} N_{i+1} = N_i + \frac{2\epsilon}{\omega^2}\sqrt{2N_i\omega}\sin\phi_i, \\ \phi_{i+1} = \phi_i + \frac{2\omega}{\epsilon_0}\sqrt{2N_{i+1}\omega} + O(\epsilon). \end{cases} \tag{3.6}$$

The preservation of phase space volume for Hamiltonian systems has a consequence that there has no attractors, that is, no subregions of lower phase-space dimension to which the motion is confined asymptotically [19]. The map given in Eq. (3.6) is the analogue of the Kepler map which was found very helpful in the hydrogen atom problem [20]. A classical analysis of mapping given in Eq. (3.6) leads to predict the onset of chaos under the same conditions found for the Standard Map description. Above the chaotic threshold a diffusive growth of "N"is observed, which we discuss in our later sections. The dimensionless standard Map Hamiltonian can be written for this system as

$$H_{sd} = \frac{p_{sd}^2}{2} + z_{sd} + \epsilon_{sd} z_{sd} \cos\tau_{sd}, \tag{3.7}$$

we use subscript "sd "for standard mapping. Here "$H_{sd} = \bar{H}_{sd}\frac{\omega_{sd}^2}{mg^2}$"are dimensionless Hamiltonian and other parameters for this Hamiltonian system are like this, "$\epsilon_{sd} = \frac{V_{sd}}{mg}$",

"$\tau_{sd} = \omega_{sd} t_{sd}$"and at the end, dimensionless Plank's Constant is "$\bar{k} = \hbar \frac{\omega^3}{mg^2}$". So we can write the map for this Hamiltonian in this form,

$$\begin{cases} \wp_{i+1} = \wp_i + K \cos \phi_i \\ \phi_{i+1} = \phi_i + \wp_{i+1} \end{cases} \tag{3.8}$$

where "$\wp_{i+1}$"and "$\wp_i$", are at the "$(i+1)th$"and "ith"kick, dimensionless momentum respectively. Now by changing these parameters we can switch from standard mapping to a conjugate mapping. The switching parameters are

$$\begin{cases} \omega = \frac{1}{\bar{k}_{sd}} \\ \epsilon = \frac{\epsilon_{sd}}{\bar{k}_{sd}^2} \\ \epsilon_0 = \frac{1}{\bar{k}_{sd}^2} \end{cases} \tag{3.9}$$

Consider the phase space defined in conjugate coordinates "(N, ϕ)"and introduce the initial distribution of phase-space points "$f(N, \phi, t = 0)$". The time evolution of "f"is described by a Fokker-Planck equation,

$$\frac{\partial f}{\partial \tau} = \frac{1}{2} \frac{\partial}{\partial N} \left(D_N \frac{\partial f}{\partial N} \right) \tag{3.10}$$

where "τ"is time measured in the number of iterations of the map, i.e. in the numbers of the bounces. The diffusion coefficient "D_N"is,

$$D_N = \frac{4\epsilon^2 N}{\omega^3}. \tag{3.11}$$

The above Fokker-Planck equation (3.10) can be solved by the method of characteristics [21].

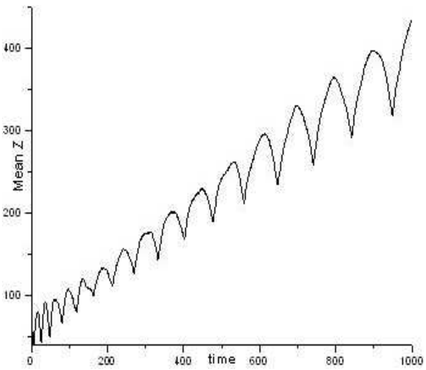

Figure 3.2: Mean Z verses time graph. In this graph we take lemda=1.7 and deltap=.5.

4 Accelerated Mapping

We define a new type of mapping as

$$\begin{cases} \wp_{i+1} = \alpha\wp_i + K(1+\alpha)\cos\phi_i \\ \phi_{i+1} = \phi_i + \wp_{i+1} \end{cases} \tag{4.1}$$

We can determine the value of "α"from the numerics as shown in Fig 2. In our calculation the average value of "α"is 5.9. Here, "α"is dimensionless quantity and it is the ratio of before and after kick velocities. From Eq. (4.1), we can see that by increasing or decreasing the value of "α", we can increase or decrease momentum of the particles. We can cool down atoms by taking "α"less then "1"and vise versa. In our numerics, we consider the "$\bar{k} = 2$", "$\lambda = 1.7$"and deltaN=.5. We consider the conjugate pair mapping as in Eq. (3.6), switching parameters and with the help of Eq. (3.10), we get the interesting relation between "$\triangle N$"and "$\bar{k}$",

$$\triangle N = \frac{4N < t > \epsilon^2}{\bar{k}} \tag{4.2}$$

We plot "$\triangle N$"verses time for different values of "$\bar{k}$"as shown in fig 3. In fig 3, we can easily see that, as we increase the value of the "$\bar{k}$", we inter into the dynamical localization window, which is "$0.24 < \lambda < \frac{\sqrt{\bar{k}}}{2}$". In this simulation, we initially take our wave packet is gaussian in position distribution as,

$$\psi(z,0) = (\frac{1}{2\pi \triangle z^2})^{\frac{1}{4}} e^{-(\frac{z-z_0}{2\triangle z})^2} \tag{4.3}$$

where "$\triangle z$"describes the width of the atomic wave packet. In order to calculate the wave function of the system after a certain time of evolution we integrate the time dependent dimensionless $Schrödiger$ wave equation. The information of the system at a later time "t"is stored in the wave function $\psi(z,t)$ which lead us to the probability distribution in the position space,

$$P(z,t) = \mid \psi(z,t) \mid^2 \tag{4.4}$$

We can find the wave function in momentum space by calculating the Fourier transform of the $\psi(z,t)$, which leads to the probability distribution in the momentum space

$$P(p,t) = \mid \psi(p,t) \mid^2 \tag{4.5}$$

after a time, t. A comparison between quantum mechanically and classical values of $\triangle N$ verses $\bar{k}$ are shown in fig 4. We take the average value of $\triangle N$ for last 400 to 1000 time, corresponding different values of the $\bar{k}$, for quantum mechanically, it shows decaying behavior and similarly corresponding $\triangle N$ verses $\bar{k}$ plot classically. Both plot shows the same behavior. We conclude by summarizing our main result. We derive a more quantum mechanically mapping for a atom bouncing off a modulated mirror under the influences of gravity. We name this mapping as conjugate pair mapping. We derive most important technique for conversion of standard map into conjugate pair map and vise versa. And then, suggest the mapping for accelerated modes and calculate the average value of α by numerically. And suggested that, we can controle momentum of the particles by controlling α. And at the end of this paper, we show the dependence of $\triangle N$ on $\bar{k}$ by numerically and by deriving the Fokker Plank equation for given system.

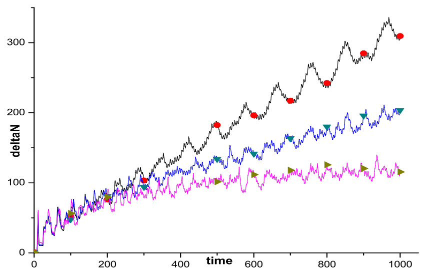

Figure 4.3: This is plot for delta N verses time for different values of "$\bar{k}$". In this graph we can easily see that if we increase the value of "$\bar{k}$"the dispersion decreases. Here, we take "$\bar{k}$ = 1"(star and line), "$\bar{k}$ = 7"(down-triangle and line) and "$\bar{k}$ = 16"(triangle and line)

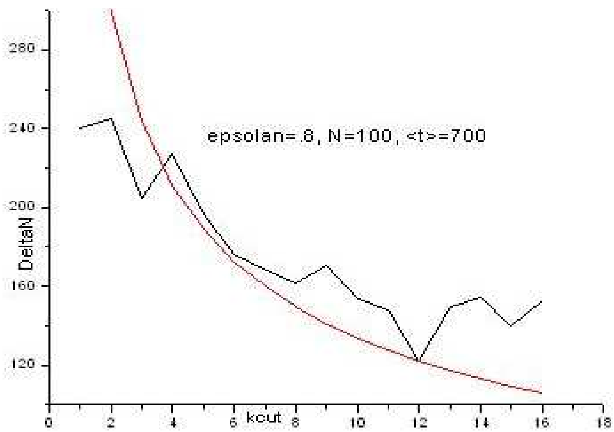

Figure 4.4: In above graph black line is for numerical value of delta P and red line is for delta P initial N=100 and blue line for delta P when initially N=110. These all lines plot delta P verses k-cut as give in the relation of eq 15.

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References

[1] G. Casati, B. V. Chirikov, F. M. Izrailev, and J. Ford, ”Stochastic Behaviour in Classical and Quantum Hamiltonian Systems, edited by G. Casati and J. Ford, Lecture Notes in Physics”, Vol. 93, Springer, Berlin, p. 334. (1979).

[2] B. V. Chirikov, F. M. Izrailev, and D. L. Shepelyansky, “Dynamical Stochasticity in Classical and Quantum Mechanics,’ Sov. Sci. Rev. C 2, 209 (1987).

[3] F. Saif, I. Bialynicki-Birula, M. Fortunato, and W. P. Schleich, “Fermi accelerator in atom optics,’ Phys. Rev. A. 58, 4779, (1998).

[4] R. Graham and M. $Höhnerbach$, “Quantum effects on the multiphoton dissociation of a diatomic molecule,’ Phys. Rev. A 43, 3966, (1991).

[5] I. Guarneri and F. Borgonovi, “Generic properties of a class of translation invariant quantum maps,’ J. Phys. A. 26, 119, (1993).

[6] F. Benvenuto, G. Casati, I. Guarneri, and D. L. Shepelyansky, “A quantum transition from localized to extended statesin a classically chaotic system,’ Z. Phys. B. 84, 159, (1991).

[7] R. Lima and D. L. Shepelyansky, “Fast delocalization in a model of quantum kicked rotator,’ Phys. Rev. Lett. 67, 1377, (1991).

[8] A. J. Lichtenterg and M. A. Liberman, Regular and Chaotic Dynamics, Springer-Verlag, NY, (1992).

[9] G. M. Zaslavsky, M. Edelman, and B. A. Niyazov, Self-similarity, renormalization, and phase space nonuniformity of Hamiltonian chaotic dynamics, Chaos 7, 159, (1997).

[10] F. Saif, “Classical and Quantum chaos in Atom Optics,’ Phys. Rep. 419, 207, (2005).

[11] M. B. d'Arcy, R. M.Godun, M. K. Oberthaler, G. S. Summy, K. Burnett, and S. A. Gardiner, Approaching classicality in quantum accelerator modes through decoherence, Phys. Rev. E. 64, 056233, (2001).

[12] S. Fishman, I. Guarneri, and L. Rebuzzini, Stable quantum resonances in atom optics, Phys. Rev. Lett. 89, 084101, (2002).

[13] R. M. Godun, M. B. d'Arcy, M. K. Oberthaler, G. S. Summy, and K. Burnett, Quantum accelerator modes: A tool for atom optics, Phys. Rev. A. 62, 013411, (2000).

[14] G. M. Zaslavsky, M. Edelman, and B. A. Niyazov, "Self-similarity, renormalization, and phase space nonuniformity of Hamiltonian chaotic dynamics' Chaos 7, 159, (1997).

[15] L. D. Pustyl'nikov, "On oscillatory motions in a certain dynamical system,' Izv. Akad. Nauk SSSR Ser. Mat 31, 325, (1988).

[16] L. D. Pustyl'nikov , "Stable and oscillating motions in nonatomic dynamical systems II,' Trudy Moskov. Mat. Obshch. 34, 3, (1977).[(English version) Trans. Moscow Math. Soc. 2, 1, (1978).].

[17] H. Wallis, "Quantum theory of atomic motion in laser light,' Phys. Rep. 255, 203, (1995).

[18] B.V. Chirikov, "A Universal Instatability OF Many-Dimensional Oscillator Systems,' Phys. Rep. 52, 263, (1979).

[19] Los. Alamos, "Hamitonian Chaos and Statistical Mechanics,' Nonlinear Science Special Issue 243. (1987).

[20] G. Casati, I. Guarneri, D. L. Shepelyansky, "Hydrogen atom in monochromatic field: chaos and dynamical photonic localization,' IEEE J. Quantum Electron. 24, 1420, (1988).

[21] H. Risken, "The Fokker-Plank Equation Methods of Solution and Applications,' Springer-Verlag Berlin Heidelberg, New York, (1996).